SEQUENCE AND SERIES

SECTION I QUESTIONS

	QUESTIONS	ANSWERS
1.	In a *jua kali* factory, the number of pans produced in the first month is 250. The number of pans produced per month increases on the average by 30. Find the expected number of pans produced for the first 12 months (3marks)	$A = 250, d = 30\ n = 12$ $S_n = \frac{n}{2}(2a + d(n-1)$ $= \frac{12}{2}(2x50 + 30(12-1)$ $6(500 + 330)$ $= 6(880)$ $= 5280pans$
2.	The sum of n terms of the sequence:3 ,9 ,15 ,21 ....is 7500.determine the value of n (2marks)	$a = 3, d = 6, n$ $S_n = \frac{n}{2}(2a + d(n-1)$ $7500 = \frac{n}{2}(2x3 + 6(n-1)$ $7500 = \frac{n}{2}(6 + 6n - 6)$ $15000 = 6n^2$ $n = \pm 50 \quad ignore\ -ve$ $\therefore n = 50$
3.	A man deposits his money in a savings bank on a monthly basis. Each deposit exceeds the previous one by sh. 750. If he started by depositing sh 1500 how much will he have deposited in 12months? (3marks)	$A = 1500, d = 750, n = 12$ $S_n = \frac{n}{2}(2a + d(n-1)$ $\frac{12}{2}(2(1500) + 750(12-1)$ $= 6(3000 + 750(11)$ $= 6 x 3000 + 8250$ $= 6x11250$ $= Sh. 67500$
4.	The average of the first and the fourth term of a GP is 140. Given that the first term is 64, find the common ratio (3marks)	$\frac{a+ar^3}{2} = 140$ $\frac{64+64r^3}{2} = 140$ $64 + 64r^3 = 280$ $64r^3 = 280 - 64$ $64r^3 = 216$ $r^3 = \frac{216}{64} \qquad r^3 = 3.375$ $r = 1.5$
5.	A machine starts production of matchboxes at the rate of 12,000 per hour. The rate of production decreases by 40% every hour. Calculate the total number of match boxes in the first two hours. (2marks)	$\frac{60}{100}x12000 = 7200$ $12000 + 7200$ $= 19,200$
6.	Onyango and Kamau were employed on the same day and their salaries were as follows: Onyango: sh 11,000 per month and an increment of sh 300 at the end of every year. Kamau: sh 10,000 per month and an increment of sh 500 at the end of every year. After how many years will they earn equal salaries (3marks)	$S_n = \frac{n}{2}(2a + d(n-1)$ $Onyango = \frac{n}{2}(2(11000) + 300(n-1)$ $Kamau = \frac{n}{2}(2x10000 + 500(n-1)$ $21700 + 300n = 19500 + 500n$ $21700 - 19500 = 500 - 300n$ $2200 = 200n$ $n = 11years$

SEQUENCE AND SERIES

7.	The first, the third and the seventh terms of an increasing arithmetic Progression are three consecutive terms of a geometric progression. If the first term of the arithmetic progression is 10 find the common difference of the A.P and the common ratio of the G.P. (4 marks)	a, a + 2d, a + 6d a, ar ar^2 $\frac{a+2d}{a} = \frac{a+6d}{a+2d}$ $a^2 + 4ad + 4d^2 = a^2 + 6ad$ $4d^2 = 6ad - 4ad$ $4d^2 = 2ad$ 4d = 2a $d = \frac{2a}{4}$ $d = \frac{2x10}{4} = \frac{20}{4} = 5$ $\therefore d = 5$ $R = \frac{10+2X5}{10} = 2$ $\therefore r = 2$
8.	An employee started on a salary of K£ 6000 per annum and received constant annual increment. If he earned, he earned a total of K£ 32,400 by the end of five years, calculate his annual increment. (3marks)	a = 600 n = 5 sum = 32400 $32400 = \frac{5}{2}(12000 + 20d)$ 64800 – 60000 + 20d 20d = 4800 d = 240
9.	The second and fifth terms of a geometric progression are 16 and 2 respectively. Determine the common ratio and the first term. (3 marks)	ar = 16, ar4 = 2 $\frac{ar^4}{ar} = \frac{2}{16}$ $r^3 = \frac{1}{8}$ $r = \frac{1}{2}$ $a = \frac{16}{0.5} = 32$
10	The first, the third and the seventh term of an increasing arithmetic progression are three consecutive terms of a geometric progression. If the first term of the arithmetic progression is 10, find the common difference of the arithmetic progression (3mks)	a, a + 2d + a + 6d 10, 10 + 2d, 10 + 6d $\frac{10+2d}{10} = \frac{10+6d}{10+2d}$ $100 + 40d + 2d^2 = 100 + 600d$ $4d^2 - 560d = 0$ $4d^2 = 560d$ $\frac{4d}{4} = \frac{560}{4}$ $d = 140$
11	In geometric progression, the first is a and the common ratio is r. The sum of the first two terms is 12 and the third term is 16. (a) Determine the ratio $\frac{ar^2}{a+ar}$ (b) If the first term is larger than the second term, find the value of r	$\frac{ar^2}{a+ar} = \frac{16}{12} = \frac{4}{3}$ $3r^2 - 4r - 4 - 0$ $3r^2 - 6r - 2r - 4 = 0$ $(3r + 2)(r - 2) = 0$ $r = \frac{-2}{3} or\ 2$ $\therefore r = \frac{-2}{3}$

SEQUENCE AND SERIES

<table>
<tr><td>12</td><td>The first three consecutive terms of a geometrical progression are 3, x and5¹/₃.Find the value of x. (2marks)</td><td>$\frac{x}{3} = \frac{16}{3x}$
$3x^2 = 48$
$x^2 = 16$
$x = 4$</td></tr>
<tr><td>13</td><td>The third and fifth term of an arithmetic progression are 10 and -10 respectively
a) Determine the first and the common difference
b) The sum of the first 15 terms</td><td>$a + 2d = 10$
$\underline{a + 4d = -10}$
$\underline{-2d = 20}$
$\mathrm{d} = -10$ $\quad a = 30$
$S_{15} = \frac{15}{2}(2x + (n - 1)\,d$
$= \frac{15}{2}(2 \times 30) + (14 \times -10)$
$= \frac{15}{2}(60 - 140) \quad = -600$</td></tr>
<tr><td>14</td><td>Find the number of terms of the series 2 + 6 + 10 + 14 + 18 + ……….. that will give a sum of 800.
.</td><td>$a = 2, d = 4$
$S_n = \frac{n}{2}[4 + (n-1)4] = 800$
$n[4 + (n-1)4] = 1600$
$4n^2 = 1600$
$n^2 = 400 \quad n = \pm 20$
$\therefore n = 20$</td></tr>
<tr><td>15</td><td>A carpenter wishes to make a ladder with 15 cross-pieces. The cross- pieces are to diminish uniformly in length from 67 cm at the bottom to 32 cm at the top. Calculate the length in cm, of the seventh cross- piece from the bottom (3 marks)</td><td>$\frac{67-32}{14} = \frac{37}{14}$
$= 2.5$
$T_7 = 67 - 6x2.5$
$= 52cm$</td></tr>
<tr><td>16</td><td>Each month, for 40 months, Amina deposited some money in a saving scheme. In the first month she deposited sh 500. Thereafter she increased her deposits by sh.50 every month.
Calculate the:
a) Last amount deposited by Amina
b) Total amount Amina had saved in the 40 months.</td><td>a). $T_{40} = 500 + (40 - 1)50$
$= 500 + 1950$
$= 2450$
b). $\mathrm{S}_{40} = \frac{40}{2}\ [500\mathrm{x}2 + (40 - 1)50]$
$= 20\ (1000 + 1950)$
$= 59000$</td></tr>
<tr><td>17</td><td>A colony of insects was found to have 250 insects at the beginning. Thereafter the number of insects doubled every 2 days. Find how many insects there were after 16 days. (3marks)</td><td>$a = 250, r = 2, n = \frac{16}{2} + 1 = 9$
$T_9 = 250x2^8$
$= 250x256$
$= 64000 \quad or$
<table><tr><td>**Days**</td><td>0</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td><td>14</td><td>16</td></tr><tr><td>**Insects**</td><td>250</td><td>500</td><td>1000</td><td>2000</td><td>4000</td><td>8000</td><td>16000</td><td>32000</td><td>64000</td></tr></table></td></tr>
</table>

SEQUENCE AND SERIES

18	A geometric progression is such that the sum of its first three terms is 14. If the first term 2, find the possible values of its common ratio.	$a = 2$ $S_3 = \frac{2(r^3-1)}{r-1} = 14$ $2r^3 - 2 = 14r - 14$ $2r^3 - 14r + 12 = 0$ $r^3 - 7r + 6 = 0$ $(r+3)(r^2 + r - 2) = 0$ $(r+3)(r+1)(r-2) = 0$ $\therefore r = -3 \; or \; -1 \; or \; 2$
19	The exterior angles of a hexagon form an arithmetic progression. If the smallest angle is15° find the size of the largest angle of the hexagon.	$a = 15$ $(a) + (a+d) + (a+2d) + (a+3d) + (a+4d) + (a+5d) += 720$ $6a + 15d = 720$ $15d = \frac{720}{6x15} = 42$ $largest\ angle = 15 + 5x42 = 225°$
20	An arithmetic progression whose first term is 2 and whose n^{th} term is 97, has the sum of its first n terms equal to 990. Find n and the common difference.	$a = 2$ $a + (n-1)d = 97 \ldots\ldots\ldots\ldots (i)$ $(n-1)d = 95$ $\frac{n}{2}[2a + (n-1)d] = 990 \ldots\ldots (ii)$ **substitute $(n-1)d = 95$ in eqn (ii)** $\frac{n}{2}[2x2 + 95] = 1980$ $99n = 1980$ $n = 20$ $using\ eqn\ (i)$ $(20-1)d = 95$ $19d = 95 \qquad d = 5$
21	The 11^{th} term of an arithmetic progression is four times the 2^{nd} term. If the sum of the first seven terms of the arithmetic progression is 175, find the first term and the common difference.	$a + 10d = 4(a + d)$ $a + 10d = 4a + 4d$ $3a = 6d \qquad a = 2d \ldots\ldots\ldots (i)$ $175 = \frac{7}{2}[2a + 6d]$ $175 = 7a + 21d \ldots\ldots\ldots\ldots (ii)$ solving the two equaions $175 = 7(2d) + 21d$ $175 = 14d + 21d$ $35d = 175$ $d = 5$ $a = 2x5 \qquad a = 10$
22	The 2^{nd}, 4^{th} and 7^{th} terms of an A.P are the first three consecutive terms of a G.P. If the common difference of the A.P is 2, find: (a) The common ratio of the G.P (b) The sum of the first 8 terms of the G.P.	a)1^{st} 3 terms of GP $a + d, a + 3d, a + 6d;$ $\frac{a+6d}{a+3d} = \frac{a+3d}{a+d}$ $(a+3d)^2 = (a+d)(a+6d)$ $a^2 + 6ad + 9d^2 = a^2 + 7ad + 6d^2$ $ad = 3d^2$ $a = 3d \qquad Given\ d = 2 \qquad a = 6$ $1st\ term\ \ 6 + 2 = 8$ $r = \frac{6+12}{12} = \frac{3}{2} \qquad r = 1.5$ ii). $S_8 = \frac{8(1.5^8 - 1)}{1.5 - 1}$ $= 394\frac{1}{16}$

SEQUENCE AND SERIES

23	In a G.P, the sum of the second and the third terms is 12, and the sum of the third and the fourth terms is -24. Find the first term and the common ratio.	$ar + ar^2 = 12 \ldots\ldots\ldots\ldots\ldots\ldots (i)$ $ar^2 + ar^3 = -24 \ldots\ldots\ldots\ldots\ldots\ldots (ii)$ $From\ eqn\ (ii)$ $r(ar + ar^2) = -24$ $12r = -24$ $r = -2$ $using\ eqn\ (i)$ $a(-2) + a(-2)^2 = 12$ $-2a + 4a = 12$ $2a = 12 \qquad a = 6$
24	The sum of the first four terms of an arithmetic progression (A.P) is 14. If the sum of the first eight terms is 108, find the sixth term of the progression.	$\frac{4}{2}[2a + 3d] = 14$ $\frac{8}{2}[2a + 7d] = 108$ $4a + 6d = 14 \ldots\ldots\ldots\ldots\ldots (i)$ $8a + 28d = 108 \ldots\ldots\ldots\ldots (ii)$ $solving\ simultaneous\ equations$ $8a + 28d = 108$ $\underline{8a + 12d = 28}$ $16d = 80$ $d = 5$ $a = \frac{108-28x5}{8} = -4$ $T_6 = -4 + 5x5 = 21$
25	The difference between the 8th term and the 4th term of an arithmetic progression is 24. The first term exceeds the common difference by 2. Find the sum of the first 10 terms of the progression	$(a + 7d) - (a + 3d) = 24$ $4d = 24$ $d = 6$ $a = 6 + 2 = 8$ $S_{10} = \frac{10}{2}[2x8 + 9x6]$ $S_{10} = 350$
26	The seventh term of an arithmetic sequence is 17. Three times the third term is 3. Calculate the first term and the common difference of the sequence.	$a + 6d = 17 \ldots\ldots\ldots\ldots\ldots\ldots (i)$ $3(a + 2d) = 3$ $3a + 6d = 3 \ldots\ldots\ldots\ldots\ldots\ldots (ii)$ $solving\ two\ equations$ $a + 6d = 17$ $\underline{3a + 6d = 3}$ $-2a = 14 \qquad a = -7$ $d = \frac{17+7}{6} = 4 \qquad d = 4$
27	The third and the tenth terms of an arithmetic progression are 11 and 39 respectively. Find: (a) The first term and the common difference of the progression. (b) The sum of the first 50 terms of the progression.	$a) \quad a + 2d = 11$ $\underline{a + 9d = 39}$ $-7d = -28$ $d = 4$ $a = 11 - (4x2) = 3$ $b) \quad S_{50} = \frac{50}{2}[2x3 + (49x4)]$ $S_{50} = 5050$

SEQUENCE AND SERIES

28	The third and the sixth terms of a geometric progression are -64 and 8 respectively. Find the common ratio and the first term of the geometric progression	$ar^2 = -64$ $ar^5 = 8$ $\frac{ar^5}{ar^2} = \frac{8}{-64}$ $r^3 = -\frac{1}{8}$ $r = -\frac{1}{2}$ $a = \frac{-64}{-0.5^2} \quad = -256$
29	The 10th, 25th and the last term of an arithmetic progression are 313, 193 and -7. Find the number of terms in the progression	$a + 9d = 313$ $a + 24d = 193$ $-15d = 120$ $d = -8$ $a = 313 + (8x9)$ $a = 385$ $last\ term\ = -7$ $-7 = 385 + (n-1) - 8$ $-392 = -8n + 8$ $-8n = -400 \qquad n = 50$
30	a) An arithmetic progression of 41 terms is such that the sum of the five terms is 560 and sum of the last terms is -250.find the first term.	$S_5 = \frac{5}{2}\{2a + (n-1)d\} = 560$ $\frac{5}{2}[2a + 4d] = 560$ $5a + 10d = 560 \ldots\ldots\ldots\ldots\ldots (i)$ $last\ five \quad T_{37}, \ldots\ldots\ldots\ldots T_{41}$ $a = T_{37} = a + 36d$ $S_5 = \frac{5}{2}\{2(a + 36d) + 4d\} = -250$ $\frac{5}{2}[2a + 72d + 4d] = -250$ $5a + 190d = -250 \ldots\ldots\ldots\ldots\ldots (ii)$ $solving\ the\ two\ simultaneous\ equations$ $5a + 10d = 560$ $5a + 190d = -250$ $180d = 810 \qquad d = -4.5$ $a = \frac{560+45}{5}$ a=121

SEQUENCE AND SERIES

SECTION II QUESTIONS

1	The first term of an arithmetic progression is 4 and the last term is 20. The sum of the term is 252. Calculate the number of terms and the common differences of the arithmetic progression (b) An Experimental culture has an initial population of 50 bacteria. The population increased by 80% every 20 minutes. Determine the time it will take to have a population of 1.2 million bacteria.	a) $\frac{n}{2}(4+20) = 252$ $N = \frac{504}{24} = 21$ $\frac{21}{2}(2x4\ (21-1)d) = 252$ $21(8+20d) = 504$ $d = \frac{16}{20} = \frac{4}{5}$ b) $50x1.8^n = \frac{1200000}{50}$ $nlog1.8 = log24000$ $n = \frac{4.3802}{0.2553} = 17.16$ $Time\ taken\ 17.16x20 = 343.2\ minutes$
2	An arithmetic progression has the first term *a* **and** the common difference *d*. a) Write down the third, ninth and the twenty fifth terms of the progression. (1mark) b) The arithmetic progression above is such that it is increasing and that the third, ninth and twenty fifth term form the first three consecutive terms of a geometric progression. The sum of the seventh and twice the sixth terms of the arithmetic progression is 78. Calculate (i) the first term and the common difference of the arithmetic Progression (5marks) (ii) the sum of the first nine terms of the arithmetic progression (2marks)	(a) $a + 2d, a + 8d, a + 24d$ (b) (i) $\frac{a+8d}{a+2d} = \frac{a+24d}{a+8d}$ $a^2 + 16ad + 64d^2 = a^2 + 26ad + 48d^2$ $16d^2 = 10ad$ $d = \frac{5a}{8}$ $2(a+5d) + (a+6d) = 78$ $3a + 16 x \frac{5a}{8} = 78$ $13a = 78$ $a = 6$ $d = \frac{5}{8} x 6 = 3.75$ (ii) $S_{10} = \frac{9}{2}\{2(6) + (9-1)3.75\}$ $= 189$
3	Abdi and Amoit were employed at the beginning of the same year. Their annual salaries in shillings progressed as follows: Abdi: 60,000, 64 800, 69, 600 Amoit 60,000, 64 800, 69 984 (a) Calculate Abdi's annual salary increment and hence write down an expression for his annual salary in his n^{th} year of employment (2 marks) (b) Calculate Amoit's annual percentage rate of salary increment and hence write down an expression for her salary in her n^{th} year of employment. (2 marks) (c) Calculate the differences in the annual salaries for Abdi and Amoit in their 7^{th} year of employment (4 marks)	a). $c.d = 64800 - 60000$ $= 69600 - 64800 = 4800$ $a = 6000$ n^{th} term $= a + (-1)\ d$ $60000 + (n-1)4800$ b). common ration $= \frac{64800}{60000} = \frac{69984}{64800} = 1.08$ n^{th} term $= ar^{n-1}$ where $a = 60000$ $r = 1.08$ $= 60000(1.08)^{n-1}$ *c)* 7th term; Abdi $= 60000 + (7-1)\ 4800$ $= 88800$ Amoit $= ar^{n-1}$ $= 60000(1.08)^6$ $= 95213$ difference $= 95213 - 88800$ $=$ sh 6413

SEQUENCE AND SERIES

4	The nth term of a sequence is given by 2n+3 a) Write the first four items of the sequence. b) Find S50, the sum of the first terms of the sequence. c) Show that the sum of the first terms of the sequence is given by. $Sn = n^2 + 4n$ Hence or otherwise find the first largest integral value of n such that. Sn < 725	a) $5, 7, 9, 11$ b) $S_{30} = \frac{50}{2}\{(2x\ 5) + (49)2\} = 2700$ c) $S_n = \frac{n}{2}(2 \times 5 + (n-1)2$ $= \frac{n}{2}(8 + 2n)$ $= 4n + n^2$ $= n^2 + 4n < 725$ $= n^2 + 4n - 725 < 0$ $(n + 2)(n - 25) < 0$ $n = 24$
5	(a) The first term of an Arithmetic Progression (AP) is 2. The sum of the first 8 terms of the AP is 156 (i) Find the common difference of the AP (ii) Given that the sum of the first n terms of the AP is 416, find n b) The 3rd, 5th and 8th terms of another AP form the first three terms of a Geometric Progression (GP). If the common difference of the AP is 3, find; (i) The first term of the GP; (ii) The sum of the first 9 terms of the GP, to 4 significant figures (2 marks)	a). i). $\frac{8}{2}[2x2 + (8-1)d] = 156$ $4(4 + 7d) = 156$ $d = 5$ ii). $\frac{n}{2}[4 + (n-1)5] = 416$ $\frac{n}{2}(4 + 5n - 5) = 416$ $5n^2 - n = 832$ $5n^2 - n - 832 = 0$ $(5n + 64)(n - 13) = 0$ $n = 13$ b). i). 1st three terms of GP are $a + 2d, a + 4d, a + 7d$ $\frac{a+4d}{a+2d} = \frac{a+7d}{a+4d}$ $(a + 4d)^2 = (a + 2d)(a + 7d)$ $a^2 + 8ad + 16d^2 = a^2 + 9ad + 14d^2$ $ad = 2d^2$ $a = 2d$ $a = 6$ $1st\ term = 6 + 6 = 12$ $r = \frac{6+12}{12} = \frac{3}{2}$ ii). $S_9 = \frac{12(1.5^9 - 1)}{1.5 - 1}$ $= 898.6\ (4\ s.f)$
6	The first term of an Arithmetic Progression (A.P.) with six terms is p and its common difference is c. Another A.P. with five terms has also its first term as p and a common difference of d. the last terms of the two Arithmetic Progressions are equal. a) Express d in terms of c. (3 marks) b) Given that the 4th term of the second A.P. exceeds the 4th term of the first one by 1 ½, find the value of c and d. (3 marks) c) Calculate the value of p if the sum of the terms of the first A.P. is 10 more than the terms of the second A.P. (4 marks)	a) $T_6 = p + 5c$ $T_5 = p + 4d$ $p + 4d = p + 5c$ $4d = 5c$ $d = \frac{5}{4}c$ b) $p + 3d - (p + 3c) = 1\frac{1}{2}$ $3d - 3c = 1\frac{1}{2}$ $\frac{5}{4}c - 3c = 1\frac{1}{2}$ $\frac{3}{4}c = \frac{3}{2}$ $c = 2$ $d = 2\frac{1}{2}$ c) $S_6 = \frac{6}{2}(24 + 5x2)$ $S6 = {}^{6}/2\ (24 + 5 \times 2)$ $S_5 = \frac{5}{2}(2p + 2\frac{1}{2})$ $= (6p + 30) - (5p + 25) = 10$ $p + 5 = 10$ $p = 5$

SEQUENCE AND SERIES

7	The first, fifth and seventh terms of an arithmetic progression (AP) correspond to the first three consecutive terms of a decreasing Geometric Progression (GP). The first term of each progression is 64, and the common difference of the AP is *d* and the common ratio of the G.P is *r*. a) (i) Write two equations involving d and r (2marks) (ii) Find the values of *d* and *r* b) Find the sum of the first 10 term of; (i) the A.P (ii) the G.P	(i)$r=\frac{64+4d}{64}$ $r=\frac{64+6d}{64+4d}$ (ii)$\frac{64+4d}{64}=\frac{64+6d}{64+4d}$ $16d^2+128d=0$ $16d(d+8)=0$ $d=-8$ $r=\frac{64+4x-8}{64}=\frac{1}{2}$ (b) (i) $S_{10}=\frac{10}{2}[2x64+9x\text{-}8]$ $=280$ (ii) $S_{10}=\frac{64[1-(0.5)^{10}]}{1-0.5}$ $=127.875$
8	The 1st, the 7th and 25th terms of an arithmetic progression are the first three consecutive terms of a geometric progression. The 20th term of the arithmetic progression is 44. Find: (a) The first term and common difference of the arithmetic progression. (3 marks) (b) The number of terms of the arithmetic progression for which the sum of the terms is 1800. (3 marks) (c) The 10th term of geometric progression. (2 marks) (d) The sum of the first n terms of the geometric progression. (2 marks)	(a) $a, a+6d, a+24d$ $\frac{a+6d}{a}=\frac{a+24d}{a+6d}$ $a^2+12ad+36d^2=a^2+24ad$ $a=3d$ $n^{th}=a+(n-1)d$ $a+19d=44$ $d=2$ $a=6$ (b)$1800=\frac{n}{2}[1x6+2n-2]$ $3600=10n+2n^2$ $n^2+5n-1800=0$ $n(n+45)-40(n+45)=0$ $n=40$ (c) $a=6, r=3$ $T_n=ar^{n-1}$ $T_{10}=6\,x\,3^9$ $=118098$ (d)$S_n=\frac{6(3^n-1)}{3-1}$ $S_n=3^{n+1}-3$
9	The product of the first three terms of a geometric progression is 64. If the first term is **a** and the common ration is **r** a) Express r in terms of a. (3 marks) b) Given that the sum of the three terms is 14; i. Find the values of *a* and *r* and hence write down two possible sequences each up to 4th term. (5 marks) ii. Find the product of the 50th term of the two sequences. (2 marks)	$=a^3r^3=64$ $r=\sqrt[3]{\frac{64}{a^3}}=\frac{4}{a}$ $=a+a.\frac{4}{a}+a.\frac{16}{a^2}=14$ $a+4+\frac{16}{a}=14$ $a^2+4a+16=14a$ $a^2-10a+16=0$ $a^2-8a-2a+16=0$ $(a-8)(a-2)=0$ $a=8 \text{ or } 2$ When $a=8$ $r=\frac{1}{2}$ $a=2$ $r=2$ 1st sequence is 8, 4, 2, 1 2nd sequence is 2, 4, 8, 16 $T_{50}=8\times\left(\frac{1}{2}\right)^{49}$ ……. 1st sequence $T_{50}=2\times(2)^{49}$ …… 2nd sequence Product = $8\left(\frac{1}{2}\right)^{49}\times 2(2)^{49}=16$

SEQUENCE AND SERIES

10	(a) In a geometrical progression the sum of the second and third term is 12 and the sum of the third and fourth terms is -36. Find the first term and the common ratio (4mks) (b) In the arithmetic progression the twelfth term is 25 and the seventh term is three times the second term find i) The first term and the common difference ii) The sum of the 1st ten terms (2mks)	a) $ar + ar^2 = 12$ $ar^2 + ar^3 = -36$ $ar(1+r) = 12$ $ar^2(1+r) = -36$ $\frac{ar^2}{ar} = \frac{-36}{12}$ $r = -3$ $ar + ar^2 = 12$ $-3a + 9a = 12$ $a = 2$ b) $a + 11d = 25 \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(i)$ $a + 6d = 3(a + d)$ $a + 6d = 3a + 3d$ $2a = 3d \ldots\ldots\ldots\ldots\ldots\ldots..(ii)$ $a = \frac{3}{2}d$ $\frac{3}{2}d + 11d = 25$ $25d = 50$ $d = 2$ $a + 11(2) = 25$ a= 3 c) $S_{10} = \frac{10}{2}\{2(3) + (10 - 2)2\}$ $= 5(6 + 18)$ = 120
11	An arithmetic progression (A.P) has the first term as a and the common difference as d. (a) Write in terms of a and d, the 3rd, 9th and 25th terms of the progression (b) The progression is increasing and the 3rd, 9th and 25th terms form the first three consecutive terms of a geometric progression (G.P). If the sum of the first 8 terms of the A.P is 153, calculate: (i) The first term and the common difference of the A.P. (4 marks) (ii)The sum of the first 20 terms of the A.P(2 marks) (iii)Find the sum of the first 5 terms of the G.P. (3marks)	**a)** $a + 2d, a + 8d, a + 24d$ **b)** $\frac{a+8d}{a+2d} = \frac{a+24d}{a+8d}$ $a^2 + 16ad + 64d^2 = a^2 + 26ad + 48d^2$ $16d^2 = 10ad$ $d = \frac{5}{8}a$ $S_8 = \frac{8}{2}[2a + (7d)] = 153$ $153 = 8a + \frac{140}{8}a$ $a = 6$ $d = \frac{5}{8}x6 = 3.75$ ii) $S_{20} = \frac{20}{2}[2x6 + (19x3.75)$ $= 832.5$ iii)G. P $r = \frac{8}{3}$ $S_5 = \frac{13.5\,x\,(\frac{8}{3}^5 - 1)}{\frac{8}{3} - 1}$ $=1084\frac{1}{6}$

SEQUENCE AND SERIES

12	Alice and Brian were employed at the beginning of the same year in a five-year contract. Their annual salaries in Shillings progressed as follows: Alice: 330000, 356400, 382800, ... Brian: 330000, 356400, 384912, ... (a) Calculate: (i) Alice's monthly salary in her fifth year of employment. (3 marks) (ii) Brian's annual percentage rate of salary increment and hence write down an expression for his monthly salary in his n^{th} year of employment. (3 marks) (b) Calculate the difference in the total income of Alice and Brian by the end of their contract. (4 marks)	a)$5^{th} = 330000 + (4x26400) = 435600$ $Monthly\ salary = \frac{435600}{12} = 36300$ b)$\frac{356400}{330000} = 1.08$ $n^{th} = ar^{n-1}$ $Monthly\ salary = \frac{330000\ x\ 1.08^{n-1}}{12}$ c)$alice\ total\ income$ $S_5 = \frac{5}{2}[2x330000 + (4x26400)$ $= 1914000$ $Brian\ total\ income$ $S_5 = \frac{330000\ x\ (1.08^5-1)}{1.08-1}$ $= 1935978.317$ $Difference = 1935978.317 - 1914000$ $= 21978.3168 \cong 21978$
13	A geometric progression (G.P.) is such that the product of its first three terms is 8,000. (a) Taking the first term as a and the common ratio as r, express r in terms of a. (2 marks) (b) The sum of the first three terms in (a) above is 78. (i)Determine the first term and the common ratio of two possible sequences. (4 marks) (ii)Hence write the first 4 terms of the two sequences. (2 marks) (c) Find the product of the 8^{th} term of the two sequences. (2 marks)	**a)** a, ar, ar^2 $8000 = a^3r^3$ $r^3 = \frac{8000}{a^3}$ $r = \frac{20}{a}$ **b)** $a + ar + ar^2 = 78$ $a + 20 + \frac{400}{a} = 78$ $a^2 - 58a + 400 = 0$ $a(a-50) - 8(a-50) = 0$ $a = 50\ or\ 8$ $when$ $a = 50 \quad r = 0.4$ $a = 8 \quad r = 2\frac{1}{2}$ ii) 8,20,50,125 50,20,8,3.2 **c)** $8^{th} = ar^7$ $50x0.4^7x8x2.5^7$ $= 400$
14	The first three terms of a geometric series are:2x, x - 8 and 2x + 5 respectively for which x is a integer. (a) Find: (i)The value of x. (3 marks) (ii)The first term and the common ratio in the series. (2 marks) (b) Calculate: (i)The value of the seventh term. (2 marks) (ii)The number of terms for which the sum is -16.625. (3 marks)	$2x, x-8, 2x+5$ $\frac{x-8}{2x} = \frac{2x+5}{x-8}$ $x^2 - 16x + 64 = 4x^2 + 10x$ $3x^2 + 26x - 64 = 0$ $x(3x+32) - 2(3x+32) = 0$ $x = 2\ or\ -32$ ii) $a = 4 \quad r = -1.5$ b) i) $7^{th} = ar^6$ $=4\ x\ -1.5^6$ $=45\frac{9}{16}$ ii)

SEQUENCE AND SERIES

15	The first term of an increasing arithmetic sequence is (2x -1) and the common difference is x. The product of the first and third term is 21. (a) Find the value of x. (3 marks) (b) If the last term of the arithmetic sequence is 105, calculate the sum of the terms. (3 marks) (c) Given that the 1st, the 4th and 13th terms of the arithmetic sequence form the first three consecutive terms of a geometric progression, find: (i)The 12th term of the geometric progression. (2marks) (ii)The sum of the first 12 terms of the geometric progression. (2 marks)	$a = 2x - 1$ $d = x$ $21 = (2x-1)(4x-1)$ $8x^2 - 6x - 20 = 0$ $2x(4x+5) - 4(4x+5) = 0$ $x = 2 \; or \; \frac{-5}{4}$ sequence 3,5,7,9 **b)**$S_n = a + (n-1)d$ $105 = 3 + (n-1)2$ $2n = 104$ $n = 52$ $S_{52} = \frac{52}{2}[6 + (51x2]$ $S_{52} = 2808$ **c)i)**3,5,7,9…… $a = 3 \qquad r = 3$ $T_{12} = ar^{11}$ $=3x3^{11} = 531441$ $S_{12} = \frac{ar^n - 1}{r-1}$ $S_{12} = \frac{3x(3^{12}-1)}{3-1}$ $= 797160$
16	The product of the first three consecutive terms at a geometric progression is 729. If the first term is a and the common ratio is r (a) Express r in terms of a. (2 marks) (b) Given that the sum of the three terms above is 39, find the values of a and r hence write down two possible sequences each up to the 4th term. (5 marks) (c) The sum of the first n terms of the sequence is 3279. Find n. (3 marks)	a, ar, ar^2 $729 = a^3r^3$ $r^3 = \frac{729}{a^3}$ $r = \frac{9}{a}$ $b) \quad a + ar + ar^2 = 39$ $a + 9 + \frac{81}{a} = 39$ $a^2 - 30a + 81 = 0$ $a(a-27) - 3(a-27) = 0$ $a = 3 \; or \; 27$ $when$ $a = 3 \qquad r = 3$ 3,9,27,81 $a = 27 \qquad r = \frac{1}{3}$ 27,9,3,1 $c) \quad S_n = \frac{3(3^n - 1)}{3-1} = 3279$ $3^n = \frac{3279x2}{3}$ $n = 7$

SEQUENCE AND SERIES

17	The sum of the first five terms of an increasing arithmetic progression (A.P) is 50 while the sum of the first six terms of the same progression is 66. (a) Find the 1st term and the common difference of the A.P. (3 marks) (b) Calculate the sum of the first 20 terms of the A.P. (2 marks) (c) The first term and two other terms of the A.P form a geometric progression (G.P). If the sum of the three terms in the G.P is 78, calculate: (i)The common ratio of the G.P. (3 marks) (ii)The sum of the first 6 terms of the G.P. (2 marks)	a) $S_n=\frac{n}{2}[2a+(n-1)d]$ $50=\frac{5}{2}[2a+(4-1)d]$ $100=10a+20d\ldots\ldots\ldots\ldots\ldots\ldots(i).$ $66=\frac{6}{2}[2a+(6-1)d]$ $132=12a+30d\ldots\ldots\ldots\ldots\ldots\ldots(ii).$ *solving the twobsimultaneous equations* $a=6 \qquad d=2$ b) $.S_{20}=\frac{20}{2}[12+(19\,x2)]$ $=500$ c)i) $S_3=\frac{6(r^3-1)}{r-1}=78$ $13(r-1)=r^3-1$ $13r-13=r^3-1$ $r^3-13r+12=0$ $(r-3)(r^2-13r+12)=0$ $(r-3)(r-1)(r+4)=0$ $r=3 \text{ or } 1 \text{ or } -4$ *since GP is increasing* $\quad r=3$ ii) $S_n=\frac{a(r^n-1)}{r-1}$ $S_6=\frac{6(3^6-1)}{3-1}$ $S_6=2184$
18	The second, sixth and eighth terms of an Arithmetic Progression (A.P) correspond to the first three consecutive terms of an increasing Geometric Progression (G.P). If the first term of the A.P is -36, the common difference of the A.P is d and the common ratio of the G.P is r: (a)(i) Write two expressions of r in terms of d. (2 mks) (ii) Find the values of d and r. (4 marks) (b) Find the sum of the first 10 terms of the G.P.(2mks) (c) Find the least number of terms of the A.P for which the sum is a positive integer. (2 marks)	**a)**$T_2=a+d \quad T_d=a+5d \quad T_8=a+7d$ $a=-36$ $r=\frac{5d-36}{d-36} \text{ or } r=\frac{7d-36}{5d-36}$ ii) $\frac{a+5d}{a+d}=\frac{a+7d}{a+5d}$ $a^2+10ad+25d^2=a^2+8ad+7d^2$ $18d^2=-2ad$ $a=-9d$ $-36=-9d$ $d=4$ **b)**$r=\frac{20-36}{4-36}=\frac{-16}{-32}=0.5$ $S_{10}=\frac{-32x(1-0.5^{10})}{1-0.5}$ $S_{10}=-63\frac{15}{16}$ **c)**$S_n=\frac{n}{2}[-72+4n-4]>0$ $\frac{n}{2}[4n-76]>0$ $n>19$ $\therefore n=20$

SEQUENCE AND SERIES

19	Three consecutive terms in a G.P are 3^{2x+1}, 9^x and 81 respectively. a. Calculate the value of x (2mks) b. Find the common ratio of the series.(2mks) c.Calculate the sum of the first 10 terms of the series. (3mks) d.Given that the 5th and 7th terms of the G.P in (a) above form the 1st two consecutive terms of an A.P Calculate the sum of the 1st 20 terms of the A.P. (3mks)	$\frac{3^4}{3^{2x}}=3^{4-2x}$ ……………………………(i) $\frac{3^{2x}}{3^{2x+1}}=3^{2x-2x-1}$ ………………………(ii) $4-2x=-1$ $x=2.5$ $b)\quad r=\frac{81}{9^{2.5}}=\frac{81}{243}=\frac{1}{3}$ $c)\quad S_{10}=\frac{729(1-\frac{1}{3}^{10})}{1-\frac{1}{3}}$ $\frac{729(0.9999)}{\frac{2}{3}}=1093.5$ $d)\quad For\ the\ AP$ $a=9, d=8\ n=20$ $s_{20}=\frac{20}{2}[18+(8x19)]$ $=1700$
20	The 2nd and 5th terms of an arithmetic progression are 8 and 17 respectively. The 2nd, 10th and 42nd terms of the A.P. form the first three terms of a geometric progression. Find (a)The 1st term and the common difference. (3mks) (b)The first three terms of the G.P and the 10th term of the G.P. (4mks) (c)The sum of the first 10 terms of the G.P. (3mks)	$a+d=8$ $a+4d=17$ $3d=9$ d = 3 $\therefore a=5$ b) $2^{nd}=8$ $10^{th}=5+9\times3=32$ $42^{nd}=5+41\times3=128$ $\therefore$ GP is 8, 32, 128, − − − a = 8 r = 4 n^{th} term of G.P $=ar^{n-1}$ $\therefore$ 10th term $=8(4^9)$ $=2097152$ (c) $S_n=\frac{a(r^n-1)}{r-1}$ $S_{10}=\frac{8(4^{10}-1)}{4-1}$ $=\frac{8}{3}\times1048575$ $=2796200$
21	The 5th and 10th terms of an arithmetic progression are 18 and -2 respectively. (a)Find the common difference and the first term(4marks) (b) Determine the list number of terms which must be added together so that the sum of the progression is negative. Hence find the sum. (6marks)	a) a + 4d = 18 $a+9d=-2$ $-5d=20$ $d=-4$ a=$[18-(4(-4))]$ a= 34 b)$s_n=\frac{n}{2}[2a+(n-1)d]<0$ $n[68+(n-1)-4]<0.$ $72n-4n^2<0$ $4n(18-n)<0$ $4n\neq0$ $18-n<0$ $n>18$ $least\ no.of\ terms=19$ $S_{19}=\frac{19}{2}\{2x34+18(-4)\}$ $S_{19}=\frac{19}{2}(-4)=-38$

SEQUENCE AND SERIES

22	The 5th term of an AP is 82 and the 12th term is 103. find: i)the first term and common difference. (3marks) ii)the sum of the 21 terms. (2 marks) b) A stair case was built such that each subsequent stair has a uniform has a uniform difference in height. The height of the 6 stair from the horizontal floor was 85 cm and the height of the 10th stair was 145. Calculate the height of the 1st stair and the uniform difference in height of the stairs. (3 marks) c)during the construction of the staircase, each step was supported by a vertical piece of timber. If the staircase has 11 stairs, calculate the total length of timber used. (2 marks)	$T_n = a + (n-1)d$ $T_5 = a + 4d = 82 \ldots\ldots\ldots\ldots (i)$ $T_{12} = a + 11d = 103$ ………………… (ii) $solving\ two\ simultaneous\ equations$ $d = 3 \qquad a = 70$ ii)$S_{21} = \frac{21}{2}[2x40 + 20x3]$ $= 2100$ b) $a + 5d = 85$ $\underline{a + 9d = 145}$ $4d = 60$ $d = 15cm$ $a + 5(15) = 85$ $a = 10cm$ **c)** $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{11} = \frac{11}{2}\{2(10) + (10)15\}$ $S_{11} = 935cm$
23	The first term of an AP is equal to the first term of a GP. The second term of an AP is equal to the fourth term of the GP while the tenth term of the AP is equal to the seventh term of the GP. (a)Given that a is the first term and d is the common difference of the AP while r is the common ratio of the GP, write the two equations connecting the AP and the GP. (2 marks) (b) Find the value of r that satisfies the progression (4marks) (c)Given that tenth term of the GP is 5120, find the values of a and d.(2marks) (d) Calculate the sum of the first 20 terms of the AP (2 marks)	**a)** .$ar^3 = a + d$ $ar^6 = a + 9d$ **b)**from (a)above $d = ar^3 - 1$ $a + 9(ar^3 - a) = ar^6$ $a + 9ar^3 - 9a = ar^6$ $ar^6 - 9ar^3 + 8a = 0$ $r^6 - 9r^3 + 8 = 0$ $(r^3 - 1)(r^3 - 8) = 0$ $r = 1\ or\ r = 2$ $r = 2$ **c)** $ar^9 = 5120$ $a = \frac{5120}{2^9} = 10$ $a + d = 10\ x\ 2^3 = 80$ $d = 80 - 10 = 70$ **d)**$s_{20} = \frac{20}{2}\{20 + (19\ x70)\}$ $= 13500$

SEQUENCE AND SERIES

24	Kubai saved sh, 2000 during the first year of employment. In each subsequent year, he saved 15% more than the preceding year until he retired. a) How much did he save in the second year? (1mark) b) How much did he save in the third year? (1mark) c) Find the common ratio between the savings in two consecutive years (1mark) d)How many years did he take to save a sum of sh 58,000? (3marks) e) How much had he saved after 20years of service (2 marks)	(a) 2nd year saving $= 2000 \text{ x } \frac{115}{100}$ $= \text{sh}. 2300$ (b) 3rd year saving $= 2300 \text{ x } \frac{115}{100}$ $= \text{sh}. 2645$ (c) common ratio $= \frac{115}{100}$ or $\frac{23}{20}$ $(d)\ 2000 \frac{(1.15^n - 1)}{1.15-1} = 58000$ $2000 \text{ x } 1.15^n = 8700 + 2000$ $1.15^n = \frac{10700}{2000}$ nlog 1.15 = log 5.35 0.0607n = 0.7284 $n = \frac{0.7284}{0.0607} = 11.99$ $= 12$ $(e)\ S_{20} = \frac{1.15^{20} - 1}{1.15 - 1}$ $= 204887.2$
25	a) Every time an insect jumps forward, the distance covered is half of the previous jump. If the insect initially jumped 8.4 cm, calculate: (i) The distance of the 8th jump. (3marks) (ii) The total distance covered after the 8th jump. (3marks) b) A ball is dropped from a height of 30 metres. Each time it strikes the ground, it bounces up to 0.8 of the previous height. How many times does the ball need to strike the ground before its bounce is less than 3 metres? (4marks)	$i)$ a = 8.4cm r = 0.5 $T_8 = 8.4(0.5)^7$ $= 0.065625$ $ii)\ S_8 = \frac{8.4x(1-0.5^8)}{1-0.5} = 16.734375$ $b) r = 0.8, a = 30$ $n^{th}\ term = ar^{n-1}$ $30x0.8^{n-1} < 3$ $0.8^{n-1} < 3$ $log0.1 > (n-1)log0.8$ $n - 1 > \frac{log0.1}{log0.8} > 10.32$ $n > 11.32$ $\therefore n = 12$
26	a) Initially, a pendulum swings through an arc of 50 cm. On each successive swing, the length of the arc is decreased by 10%. After 10 swings, what total length will the pendulum have swung? (3marks) b) The 1st ,2nd and 3rd terms of the sequence are given as {4+5+6}, {7+8+9} and {10+11+12} respectively. Find the 20th term. (3marks) c) Find the sum of the series -22+-19+-16+……. +239 (4marks)	$a)\ a = 50, r = 0.9 \quad n = 10$ $S_{10} = \frac{50(1-0.9^{10})}{1-0.9}$ $Total\ Length = 325.66078$ $b)\ a = 15 \quad d = 9 \quad n = 20$ $T_{20} = 15 + (19x9)$ $T_{20} = 186$ $\{x, x+1, x+2\} = 186$ $\therefore\ T_{20} = \{61 + 62 + 63\}$ $c)\ a = -22, d = 3\ \ last\ term = a + (n - 1)d$ $239 = -22 + 3n - 3 \quad n = 88$ $S_{88} = \frac{88}{2}\{2x - 22 + (87x3)\}$ $S_{88} = 9548$

SEQUENCE AND SERIES

27	a) The first term of an AP and GP is 4. The common ratio of the GP is 8 less than the common difference of the AP. The ratio of the third term of the AP to the third term of the GP is 7:16. Find the value of the common difference and the common ratio. (4 marks) b) A ball falls vertically from a height of 15m. Each time it bounces back to 50% of the height achieved on the previous bounce. Find the distance covered after 6 such bounces. (3marks) c) Find the number of terms in the following sequence 8, 4, 2, ½ ……., $^1/_{512}$ (3marks)	a) $a = 4$ $d - r = 8 \ldots\ldots\ldots\ldots (i)$ $d = 8 + r$ $\frac{a+2d}{ar^2} = \frac{7}{16}$ $16a + 32d = 7ar^2 \ldots\ldots\ldots\ldots (ii)$ $64 + 32d = 28r^2$ $subt\ d = 8 + r \ \ in\ eqn\ (ii)$ $64 + 32(8 + r) = 28r^2$ $64 + 256 + 32r = 28r^2$ $7r^2 - 8r - 80 = 0$ $r = \frac{8 \pm \sqrt{64^2 - (4x7x80)}}{14}$ $r = \frac{8 \pm 48}{14} = 4\ or\ -2.857 \quad ignore\ -ve\ value$ d = 8 + 4 = 12 d = 12 $r = 4$ **b)** $a = 15, r = 0.5$ $S_6 = \frac{15(1-0.5^6)}{1-0.5}$ = 29.5314metres **c)** nth term is ar^{n-1} a = 8, r = ½ nthterm = $\frac{1}{512}$ $8(\frac{1}{2})^{n-1} = \frac{1}{512}$ $8(\frac{1}{2})^{n-1} = 2^{-9}$ $(\frac{1}{2})^{n-1} = \frac{2^{-9}}{8}$ $(\frac{1}{2})^{n-1} = 2^{-12}$ $(\frac{1}{2})^{n-1} = (\frac{1}{2})^{12}$ $n - 1 = 12$ $n = 13$

SEQUENCE AND SERIES

29	a) A geometric series is such that its first term is 2. Find the two common ratios if the sum of its first three is 26. (3marks) b) The product of the third and the sixth terms of an arithmetic sequence is 406.The ninth of the sequence divided by the fourth term gives a quotient of 2 and a remainder of 6. Find the first term and the common difference of the arithmetic sequence.	$a)\quad a=2$ $S_3=\frac{2(r^3-1)}{r-1}=26$ $2r^3-2=26r-26$ $2r^3-26r-24=0$ $r^3-13r+12=0$ $(r-3)(r^2-13r+12)=0$ $(r-3)(r-1)(r+4)=0$ $\therefore r=3$ $b)\ T_3=a+2d \qquad T_6=a+5d$ $\frac{a+8d}{a+3d}=2\ rem\ 6$ $(a+2d)(a+5d)=406$ $a^2+7ad+10d^2=406\ldots\ldots\ldots\ldots\ldots(i)$ $a+8d=2(a+3d)+6$ $a+8d=2a+6d+6$ $\mathrm{a}-2\mathrm{d}=-6\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\mathrm{ii})$ $a=2d-6$ $subt\quad a=2d-6\ in\ eqn\ (i)$ $(2d-6)^2+7d(2d-6)+10d^2=406$ $4d^2-24d+36+14d^2-42d+10d^2=406$ $28d^2-66d-370=0$ $d=\frac{66\pm\sqrt{66^2-(4x28x-370)}}{2x28}$ $d=\frac{66\pm214}{56}$ $d=5\ or\ -2.643\quad ingnore\ -ve \qquad \therefore d=5$ $a=2x5-6 \qquad\qquad a=4$

NOTE:

- The n^th term of an A.P: $T_n=a+(n-1)d$ where $\boldsymbol{a}$ is the first term and $\boldsymbol{d}$ is the common difference.
- The sum of the first $\boldsymbol{n}$ terms in an arithmetic series: $S_n=\frac{n}{2}[a+L]$ where $l=T_n$ hence $S_n=\frac{n}{2}[2a+(n-1)d]$
- The n^th term geometric progression (G.P): $T_n=ar^{n-1}$ where $\boldsymbol{a}$ is the first term and $\boldsymbol{r}$ common ratio.
- The sum of the first $\boldsymbol{n}$ terms in geometric series: $S_n=\frac{a(r^n-1)}{r-1}; r>1$ or $S_n=\frac{a(1-r^n)}{1-r}; r<1$